

Exercise 43

Find the limit.

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{5x^3 - 4x}$$

Solution

Rewrite the limit in terms of ones that are known.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin 3x}{5x^3 - 4x} &= \lim_{x \rightarrow 0} \frac{\sin 3x}{x(5x^2 - 4)} = 3 \lim_{x \rightarrow 0} \left(\frac{\sin 3x}{3x} \cdot \frac{1}{5x^2 - 4} \right) \\ &= 3 \left(\lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \right) \left(\lim_{x \rightarrow 0} \frac{1}{5x^2 - 4} \right) \\ &= 3 \left(\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \right) \left(\lim_{x \rightarrow 0} \frac{1}{5x^2 - 4} \right) \\ &= 3(1) \left(-\frac{1}{4} \right) \\ &= -\frac{3}{4} \end{aligned}$$